**Signals & Systems**

**EEE-223**

Lab # 05



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**LAB # 05**

**Study of Properties of Systems (Linearity, Causality, Memory, Stability and Time invariance)**

**Lab 05-** **Study of Properties of Systems (Linearity, Causality, Memory, Stability and Time invariance)**

**Pre-Lab Tasks**

**5.1 Properties of Systems:**

**5.1.1 Causal and Non-causal Systems:**

A system is causal if the system output at time  does not depend on values of the input for. In other words, for any input signal, the corresponding output depends upon the present and past values of. So, if the input to a causal system is zero for, the output of the system is also zero for. Correspondingly a discrete time system is causal if its output at time depends only on the values of input signalfor. All natural systems are causal.

**Example:**

Suppose the systemis described by the I/O relationship while the I/O relationship of the system is given by . Using the input signal find out if the two systems are causal.

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| Commands | Results | Comments |
| t1=-3:0.1:0;  x1=zeros(size(t1));  t2=0:0.1:1;  x2=ones(size(t2));  t3=1:0.1:3;  x3=zeros(size(t3));  t=[t1 t2 t3];  x=[x1 x2 x3];  plot(t,x,'linewidth',2),grid on  ylim([-0.1 1.1]);  legend('x(t)') | causalnoncausal1.bmp | Definition of the graph in the time interval of the input signal |
| plot(t-1,x,'linewidth',2),grid on  ylim([-0.1 1.1]);  legend('y\_1(t)') | causalnoncausal2.bmp | The output of is given by. The input signal is zero for but the output is nonzero for , i.e., depends upon the future values of; thus system  is non-causal |
| plot(t+1,x,'linewidth',2),grid on  ylim([-0.1 1.1]);  legend('y\_2(t)') | causalnoncausal3.bmp | The output ofis given by. The output is zero for , i.e., depends only on the past values of ; thus system  is causal |

**5.1.2 Static (Memory less) and Dynamic (with Memory) Systems:**

A system is static or memory less if for any input signalorthe corresponding outputordepends only on the value of the input signal at that time. A non-static system is called dynamic or dynamical.

**Example:**

Using the input signal find out the systems described by the I/O relationship and are static or dynamic.

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| Commands | Results | Comments |
| t1=-3:0.1:0;  x1=zeros(size(t1));  t2=0:0.1:1;  x2=ones(size(t2));  t3=1:0.1:3;  x3=zeros(size(t3));  t=[t1 t2 t3];  x=[x1 x2 x3];  plot(t,x,'linewidth',2),grid on  ylim([-0.1 1.1]);  legend('x(t)') | causalnoncausal1.bmp | Definition of the graph in the time interval of the input signal |
| plot(t,3\*x,'linewidth',2),grid on  ylim([-0.1 3.1]);  legend('y(t)') | staticnonstatic2.bmp | The output of the system when I/O relationshipdepends on the value of the input at the same time. Hence, it is a static (or memory less) system |

In order to determine if the second system described by the I/O relationshipis static or dynamic, recall thatthus and so. The values of depend on past values of so system is dynamic.

**5.1.3 Linear and Non-linear Systems:**

Letdenote the response of the systemto an input signal, that is, . System  is linear if for any input signal sand and any scalarand the following relationship (equation 5.1) holds.



In other words, the response of the linear system to an input that is a linear combination of two signals is the linear combination of the responses of the system to each one of these signals. The linearity property is generalized for any number of input signals, and this is often referred to as the principle of superposition. The linearity property is the combination of two other properties: the additivity property and the homogeneity property. A system satisfies the additivity property if for any input signalsand 



While the homogeneity property implies that for any scalarand any input signal,



**Example:**

Letand be the input signals to the systems described by the I/O relationships  and. Determine if the linearity property holds for these two systems.

To examine if the systems are linear, we use the scalars and The time interval considered is.

For the system described by the I/O relationship the procedure is followed as

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| Commands | Results | Comments |
| t=-3:0.1:3;  x1=heaviside(t)-heaviside(t-1);  x2=heaviside(t)-heaviside(t-2); |  | Definition of the input signals and |
| %computation of the left side of equation 5.1  a1=2;  a2=3;  z=a1\*x1+a2\*x2; |  | The expression is defined. |
| y=2\*z;  plot(t,y,'linewidth',2),grid on  ylim([-1 11]) | linearnonlinear1.bmp | The left side of equation 5.1, namely, is computed and the result is plotted. |
| %computation of the right side of equation 5.1  z1=2\*x1;  z3=3\*x2; |  | Definition of  and. |
| y=a1\*z1+a2\*z2;  plot(t,y,'linewidth',2),grid on  ylim([-1 11]) | linearnonlinear1.bmp | The right side of equation 5.1, namely, is computed and the result is plotted. |

The two graphs obtained are identical; hence the two sides of equation 5.1 are equal. Therefore the system described by the I/O relationshipis linear.

**5.1.4 Time-Invariant and Time-Variant Systems:**

A system is time invariant, if a tune shift in the input signal results in the same time shift in the output signal. In other words, ifis the response of a time-invariant system to an input signal, then the system response to the input signal is. The mathematical expression (equation 5.2) is



Equivalently, a discrete time system is time or (more appropriately) shift invariant if



From the above equations, we conclude that is a system is time invariant, the amplitude of the output signal is the same independent of the time instance the input is applied. The difference is time shift in the input signal. A non-time invariant system is called time-varying or time-variant system.

**Example:**

Suppose that response of a system to an input signal. Determine if the system is time invariant by using the input signal.

In order to determine if the system is time invariant, first we will have to compute and plot the system responseto the given input signal. Next, the computed output is shifted by 3 units to the right to represent the signal. This corresponds to the left side of equation 5.2. As for the right side if equation 5.2, first the input signal is shifted 3 units to the right in order to represent the signal. Next, the system responseis computed and plotted. If the two derived system responses are equal, the system under consideration is time invariant.

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| Commands | Results | Comments |
| t=-5:0.001:10;  p=heaviside(t)-heaviside(t-5);  y=t.\*exp(-t).\*p;  plot(t,y,'linewidth',2),grid on  ylim([-0.05 0.4])  legend('y(t)') | timvariantinvariant1.bmp | The responseof the system to the input signalis , |
| plot(t+3,y,'linewidth',2),grid on  ylim([-0.05 0.4])  legend('y(t-3)') | timvariantinvariant2.bmp | The output signalis shifted 3 units to the right in order to obtain the signal. |

The input signal is given by Thus the system response is computed as.

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| Commands | Results |
| t=-5:0.001:10;  p=heaviside(t-3)-heaviside(t-8);  y=t.\*exp(-t).\*p;  plot(t,y,'linewidth',2),grid on  ylim([-0.01 0.2])  legend('S[x(t-3)]') | timvariantinvariant3.bmp |

The two obtained graphs are not alike; thus the system described by the I/O relationship is time variant. A rule of thumb is that if the output of the system depends on time outside of the system is time variant.

**5.1.5 Invertible and Non-Invertible Systems:**

A system is invertible if the input signal that is applied to the system can be derived from the system response. In other words, a system is invertible if the I/O relationship is one to one, namely if different input values correspond to different output values.

**Example:**

Determine if the systems and described by the I/O relationships and, respectively, are invertible. Consider the signal as the input signal.

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| Commands | Results | Comments |
| n=-2:2;  x=2\*n; | x= -4 -2 0 2 4 | Input signal |
| y1=3\*x; | y1= -12 -6 0 6 12 | The output signal of the system |
| y2=x.^2; | y2= 16 4 0 4 16 | The output signal of the system |

**5.1.6 Stable and Unstable Systems:**

Stability is very important system property. The practical meaning of a stable system is that for a small applied input the system response is also small (does not diverge). A more formal definition is that a system is stable or bounded-input bounded-output (BIBO) stable if the system response to any bounded-input signal is bounded-output signal. The mathematical expression is as follows: Suppose that a positive number exists, such that. The system is stable if a positive number exists, such that. A non-stable system is called unstable system.

**Example:**

Suppose that input signalis applied to two systems described by the I/O relationships and. Determine if the two systems are stable.

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| Commands | Results | Comments |
| 1t=0:0.01:10;  x=cos(2\*pi\*t);  plot(t,x,'linewidth',2), grid on  ylim([-2 2]) | stableunstable1.bmp | Definition and graph of . The input signal is bounded as namelyis bounded by as . |
| y1=x.^2;  plot(t,y1,'linewidth',2), grid on  ylim([-0.5 1.5]) | stableunstable2.bmp | Definition of graph of. The out signalis bounded as namelyis bounded by, asHence the system described by the I/O relationshipis BIBO stable. |
| y2=t.\*x;  plot(t,y2,'linewidth',2), grid on | stableunstable3.bmp | Definition of the graph of. The output signalis not bounded as its amplitude is getting larger as time passes. Hence the system with I/O relationshipis not BIBO stable. |

**In-Lab Tasks**

**Task 01: Find out if the discrete-time system described by the I/O relationship  is:**

1. Static or Dynamic (input signal )
2. Causal or non-causal (input signal)
3. Linear or non-linear (input signals )
4. Shift invariant or shift variant (input signaland shift)

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| n = -2:2;  x = 2.\*n;  subplot(2,1,1)  stem(n,x,'LineWidth',2)  xlim([-2.5,2.5])  ylim([-4.5,4.5])  title('y[n] = x[n]')    y = -x;  subplot(2,1,2)  stem(n,y,'LineWidth',2)  xlim([-2.5,2.5])  ylim([-4.5,4.5])  title('y[n] = x[-n]')  A picture containing graphical user interface  Description automatically generated   1. The system is dynamic.(with memory) 2. The system is Non-causal as the output depends on the future value of input.   n = -2:4;  x1 = 2.\*n;  x2 = n./3;  a1 = 2;  a2 = 3;  x = a1.\*x1 + a2.\*x2;  y = -x;  subplot(2,1,1)  stem(n,y,'LineWidth',2)  xlim([-2.5,4.5])  ylim([-22,12])  xlabel('n')  ylabel('y1[n]')    y1 = -x1;  y2 = -x2;  y = a1.\*y1 + a2.\*y2  subplot(2,1,2)  stem(n,y,'LineWidth',2)  xlim([-2.5,4.5])  ylim([-22,12])  xlabel('n')  ylabel('y2[n]')  Graphical user interface  Description automatically generated with medium confidence   1. As both the graphs are identical, hence the system is said to be linear.(as both the side of equation are equal)   n = -2:4;  x = 2.\*n;  n0 = 3;  subplot(3,1,1)  stem(-n,x,'LineWidth',2)  legend('y[n]')  subplot(3,1,2)  stem(-(n-3),x,'LineWidth',2)  legend('y[n+3]')  x = 2.\*(n+3);  subplot(3,1,3)  stem(-n,x,'LineWidth',2)  legend('S(x[n-3])')  **Diagram, calendar  Description automatically generated**   1. as the graphs are not alike hence system is time variant |

**Task 02: Find out if the discrete-time system described by the I/O relationship **

**is:**

1. Static or Dynamic (input signal )
2. Causal or non-causal (input signal)
3. Linear or non-linear (input signals )
4. Shift invariant or shift variant (input signaland shift)

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| n = -2:2;  x = 2.\*n;  subplot(2,1,1)  stem(n,x,'LineWidth',2) %print x[n]  xlim([-2.2 2.2])  ylim([-4.5 4.5])  legend('x[n]')  xlabel('n')    y = x;  subplot(2,1,2)  stem(1-2.\*n,y,'LineWidth',2) %print y[n]  xlim([-3.2 5.2])  ylim([-4.5 4.5])  legend('y[n]')  xlabel('n')  A picture containing chart  Description automatically generated   1. System is dynamic( with memory). 2. As the system depends on future as well as past values hence it is non causal.   n = -2:4;  x1 = 2.\*n;  x2 = n./3;  a1 = 2;  a2 = 3;  x = a1.\*x1 + a2.\*x2;  subplot(2,1,1)  stem(1-2.\*n,x,'LineWidth',2)  legend('a1.x1[n] + a2.x2[n]')    legend('a1.x1[n] + a2.x2[n]')  subplot(2,1,2)  y1 = a1.\*x1;  y2 = a2.\*x2;  y = y1 + y2;  stem(1-2.\*n,y,'LineWidth',2)  legend('a1.y1[n] + a2.y2[n]')  A picture containing timeline  Description automatically generated   1. As both the graphs are same, hence the system is linear.   n = -2:4;  x = 2.\*n;  n0 = 3;  subplot(2,1,1)  stem(n0+1-2.\*n,x,'LineWidth',2)  legend('S[n-3]')    y = 2.\*(n-n0);  subplot(2,1,2)  stem(1-2.\*n,y,'LineWidth',2)  legend('S[x[n-3]]')  Timeline  Description automatically generated with medium confidence   1. As the graphs are not alike so the system is time variant. |

**Post-Lab Task**

## Critical Analysis / Conclusion

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| In this lab, we studied and practically implemented the concepts of Linearity, Causality, Memory, Stability and Time Variance. We completed various tasks and leant to differentiate the properties of these concepts using their graphs by plotting them on MATLAB. |

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| **Lab Assessment** | | |
| **Pre-Lab** | **/1** | **/10** |
| **In-Lab** | **/5** |
| **Critical Analysis** | **/4** |
| **Instructor Signature and Comments** | | |